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A simplified determination of the *J*-integral for paper

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A novel, simplified, method for evaluating the *J*-integral for paper sheets has been developed. The inaccuracy of the single-specimen method (proposed by Rice *et al.* [8]), when applied to the fracture toughness testing of paper, was found to result from an erroneous assumption about the relationship between the load and the displacement due to plasticity. A new method was developed theoretically; it is based on a modified assumption about this relationship. Using this method, the *J*-integral can be evaluated with a single load–displacement curve together with a new parameter that must be separately evaluated using at least one additional specimen. Although the new method requires a minimum of two notched specimens, it provides more flexibility than previous modifications of the single-specimen method. The experimental study indicated that the present method yields values of J_c (the critical value of the *J*-integral) which agree very closely with those obtained using the cumbersome multiple-specimen method, and is therefore more suitable for paper testing than other methods presently available.

1. Introduction

For the past twenty years, scientists studying the phenomenon of paper fracture have attempted to apply the principles of fracture mechanics in order to understand the relationship between crack-growth resistance and paper structure. The conventional test used to measure work of fracture is the Elmendorf tear test, but there are a number of fundamental objections to this test, and it is primarily used as a quality-control test in mill environments. Fracture mechanics has been recognized as a powerful tool which can be used to characterize the ability of a sheet to resist crack-growth under conditions similar to those which might be found in practice, for example, on printing presses.

In early studies, linear elastic fracture mechanics (LEFM) was applied directly to various types of paper [1]. It is well known, however, that large amounts of plastic deformation are observed during tensile tests of paper specimens, particularly for specimens cut with their long axis oriented in the cross-machine direction of anisotropic machine-made sheets (Fig. 1). Uesaka *et al.* [2] showed that the plastic-zone sizes defined by McClintock and Irwin [3] in double-edge-notched (DEN) paper specimens can be relatively large when compared to the crack length.

Uesaka *et al.* [2, 4] addressed the shortcomings of LEFM by applying *J*-integral analysis to the evaluation of the fracture toughness of paper. The *J*-integral was proposed by Rice as a means of characterizing the stress–strain singularity at a crack tip in an elastic or elastic–plastic (small-scale yielding) material [5]. A critical value of the *J*-integral, J_c, is commonly used to characterize ductile fracture in materials exhibiting large-scale yielding [6–10]. Although the *J*-integral is not equal to the energy available for crack growth in materials which deform plastically, a large number of experimental results [6, 7, 11–13] and numerical calculations [14–17] have shown that the criterion $J = J_c$ can be used to predict fracture in large-scale yielding materials.

Two standard methods of evaluating the *J*-integral have been applied to the testing of paper sheets [4]: the multiple-specimen method developed by Begley and Landes [6], and the single-specimen method developed by Rice *et al.* [8]. The multiple-specimen method is based on the fundamental meaning of the *J*-integral and does not contain any simplifying assumptions. Unfortunately, a large number of specimens are needed to evaluate the *J*-integral using this method. The single-specimen method of Rice *et al.* was derived using a simple assumption about the plastic deformation in a specimen exhibiting large-scale yielding and requires considerably less experimental effort.

Perhaps one of the most important requirements of a fracture-toughness test is that it should yield a toughness parameter which is a true material property, and is thus independent of specimen geometry. Matoba [19] and Uesaka [4] have investigated the effect of notch length on J_c for paper and have found higher J_c values for specimens with shorter notch lengths. They suggested that the higher values may

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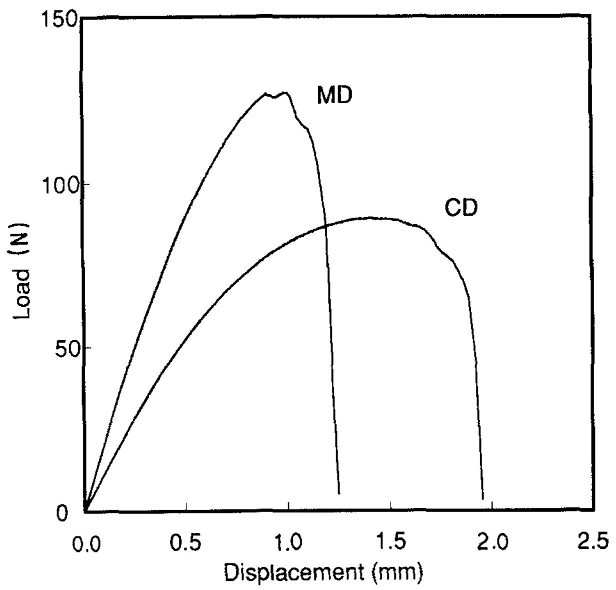

Figure 1 Typical load–displacement curves for DEN specimens of commercial fine paper having a notch length of 20 mm: MD; machine direction; and CD; cross-machine direction.

have been a result of latent, stable, crack growth. The onset of crack growth must be determined precisely for a proper evaluation of J_c, however this is one of the principal experimental difficulties in the fracture-toughness testing of paper. Direct observation of crack growth in paper is extremely difficult [20, 21], and this might have led to the specimen geometry dependence of J_c observed previously.

Because it is relatively easy to perform, the single-specimen method has been applied to paper fracture in many studies. Since the multiple-specimen method of Begley and Landes is based directly on the definition of the J-integral, it is reasonable to assume that it produces the "correct" value of J (within experimental error), and that other, simplified, methods should produce a value in agreement with this. In two studies of paper fracture, Steadman and Fellers [20] and Pouyet *et al.* [22] reported that the results of the single-specimen method did not agree with the values of J_c obtained using the multiple-specimen method. In other studies, Matoba [19] and Uesaka [4] showed that the difference between the fracture toughness of filter paper determined by the single- and multiple-specimen methods was within the level of experimental error. Recently, Westerlind *et al.* [21] reported (in their study of a non-linear technique for evaluating the J-integral for liner board) that the values estimated using the single-specimen method were larger than the values produced by the multiple-specimen method. The discrepancy between the results of the two methods seems to be a function of material properties. Although the accuracy of the single-specimen method for estimating the J-integral for metals and general strain-hardening materials has been studied by many authors [15, 18, 23–38], a detailed analysis of the accuracy of the method for film-like thin materials or paper sheets has not been presented.

In this study, the accuracy of the simplified equation (the single-specimen method) for evaluating the J-integral in paper sheets is examined in detail. The disagreement between the values obtained with the single-specimen method and those obtained using the multiple-specimen method is discussed first, and a very simple modification of the single specimen method is developed theoretically and then examined experimentally. The modified single-specimen method produces results which agree very well with those of the multiple-specimen method but the experimental effort required to evaluate the J-integral is kept to a minimum.

2. Evaluation of the J-integral

2.1. Definition of the J-integral

Rice [5] developed the path-independent energy line integral which is commonly called the J-integral, as an energy-based parameter characterizing the stress–strain field near a crack tip surrounded by small-scale plasticity. The J-integral is defined as

$$J = \int_{\Gamma} \left(W\,\mathrm{d}y - \boldsymbol{T}\frac{\partial \boldsymbol{u}}{\partial x}\,\mathrm{d}s \right) \tag{1}$$

where W is the strain energy density, and $\boldsymbol{T}$ is a traction vector defined normal to a contour, Γ, surrounding the crack tip. Rice [5] and Begley and Landes [6] showed that J-integral can be interpreted as the rate-of-change of potential energy per unit crack length

$$J = -\frac{\mathrm{d}U}{\mathrm{d}a} = \int_0^{\delta} \left(-\frac{\partial P}{\partial a} \right)_{\delta} \mathrm{d}\delta = \int_0^{P} \left(\frac{\partial \delta}{\partial a} \right)_{P} \mathrm{d}P \tag{2}$$

where U is the potential energy per unit thickness, a is the crack length, P is the load per unit thickness and δ is displacement. U can be measured by using the area under a load–displacement curve; however, for large-scale yielding materials like paper sheets, U does not represent the energy available for crack growth but rather the work done in deforming the specimen.

2.2. Multiple-specimen method [6]

The multiple-specimen method is based directly on the interpretation of the J-integral expressed by Equation 2. A schematic outline of the approach is presented in Fig. 2. Using load–displacement curves for a series of notched specimens, the work done up to a particular value of displacement is evaluated for each notch length (Fig. 2a). After drawing an energy versus notch-length curve for each displacement, the slope of these curves may be interpreted as the J-integral (Fig. 2b). The J-integral value for each displacement is obtained by dividing the slope by the sheet thickness. Drawing the J-integral versus displacement curve, the critical value of J-integral is estimated using the displacement of the onset of crack growth (Fig. 2c).

The multiple-specimen method requires at least three specimens with differing notch lengths to obtain one J-integral value. In addition, because paper is an inherently variable material, a number of specimens must be tested to have reliable load–displacement curves for the sample (the number depends on the uniformity of the sample). The single-specimen

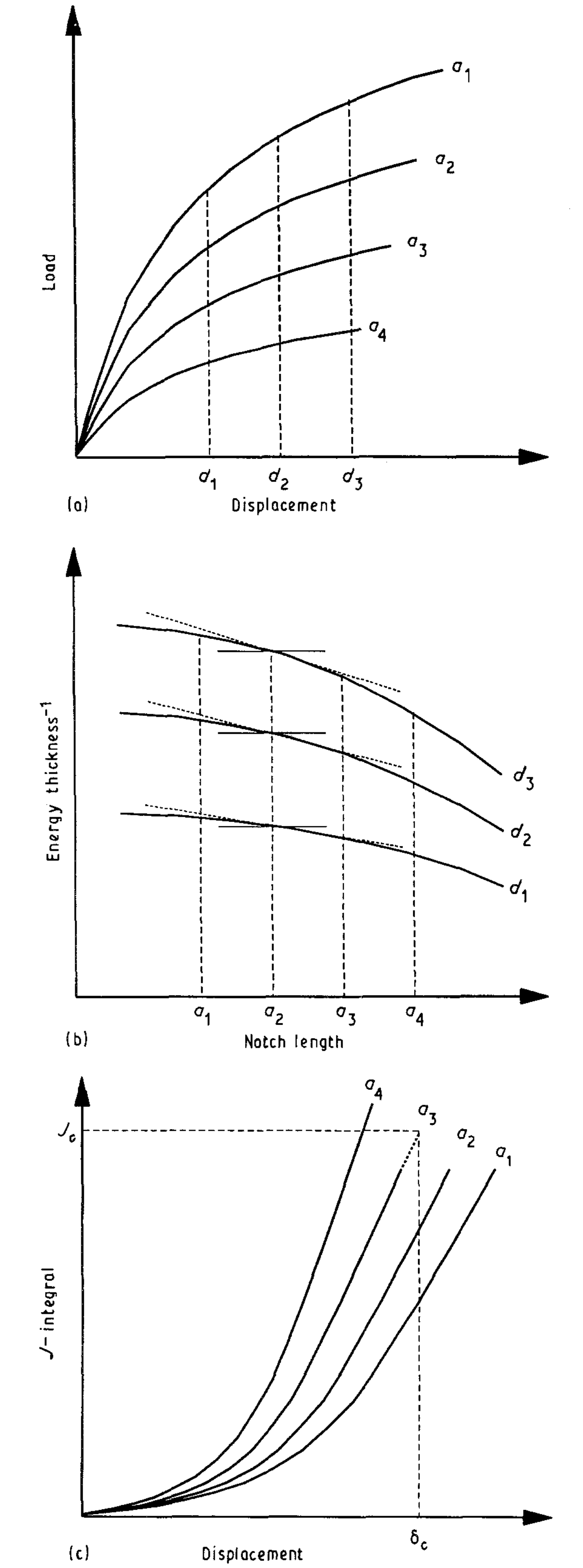

Figure 2 Schematic showing the implementation of the multiple specimen-method for J-integral estimation.

method, which yields one J_c value per specimen, was developed to reduce this experimental burden.

2.3. Single-specimen method [8]

The single-specimen method was derived on the basis of two assumptions. The total displacement, δ, is assumed to be the sum of the displacements due to elasticity and plasticity

$$\delta = \delta_e + \delta_p \tag{3}$$

In addition, the plastic displacement is assumed to have the following form

$$\delta_p = b\,h(P/b) \tag{4}$$

where b is the ligament width and h is a function of P/b involving material parameters E (elasticity), σ_y (yield stress), and n (strain-hardening coefficient), as described in the original paper by Rice *et al.* [8].

Using these assumptions the J-integral can be written as the sum of the J-integral related to elastic and plastic displacements

$$J = J_e + J_p$$

$$= J_e + \int_0^P \left(-\frac{\partial \delta_p}{\partial b}\right)_P \mathrm{d}P \tag{5}$$

$$= J_e + \frac{1}{b}\left(2\int_0^{\delta_p} P\,\mathrm{d}\delta_p - P\delta_p\right) \tag{6}$$

where J_e is equal to the strain-energy release rate, G, which is given for isotropic materials as

$$J_e = G = \frac{K^2}{E} \tag{7}$$

where K is the stress intensity factor and E is Young's modulus. For orthotropic materials, this relationship becomes

$$J_e = G = K_I^2 = \frac{1}{(2E_xE_y)^{1/2}}\left[\left(\frac{E_x}{E_y}\right)^{1/2} + \frac{2E_x}{E_{45}} - \frac{E_x + E_y}{2E_y}\right]^{1/2} \tag{8}$$

where E_x and E_y are the moduli in the x (parallel with the notch), and y (perpendicular to the notch, parallel to the loading direction) directions and E_{45} is the modulus 45 degrees from the notch direction. K_I is the stress-intensity factor for the in-plane crack-opening mode, which can be determined for the DEN specimen as

$$K_I = \frac{P}{w}(\pi a)^{1/2} F(a/w) \tag{9}$$

where w is the specimen width, a is the notch length and $F(a/w)$ is a finite-width correction factor [39].

In practical measurements using the single-specimen method, the modulus (or moduli in the orthotropic case) must be measured using an unnotched specimen(s) so that J_e can be related to the specimen geometry according to Equation 9 and either Equation 7 or 8. The plastic part of the J-integral, J_p, can be determined by using a load–displacement curve: the integrand in Equation 5 is equal to the twice the shaded area under the curve in Fig. 3 [4]. Although the single-specimen method requires at least one notched and one unnotched specimen in practice, it nevertheless requires much less experimental effort than the original multiple-specimen method.

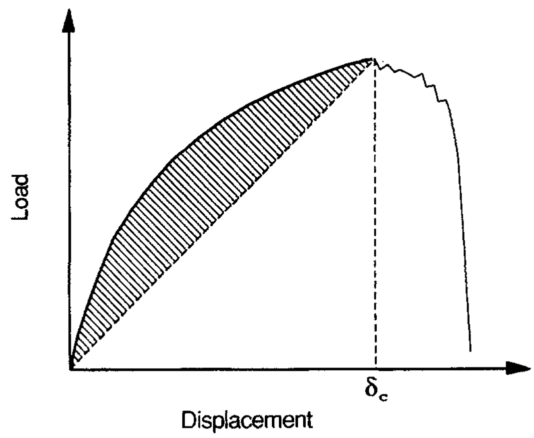

Figure 3 Schematic load–displacement curve for a notched specimen.

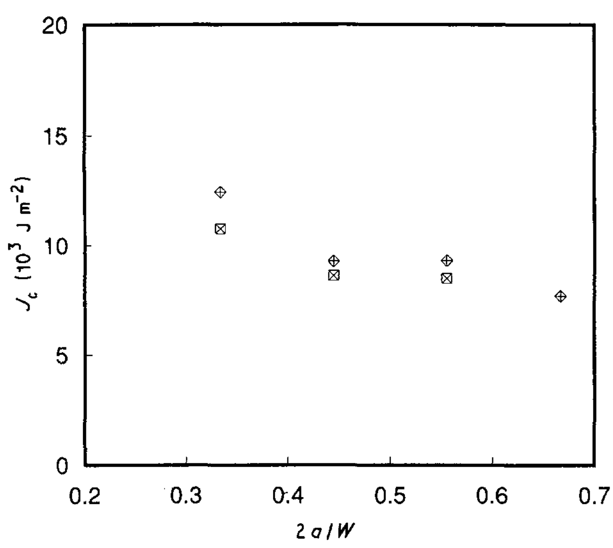

Figure 4 Critical J-value, J_c, against notch length normalized by specimen width for specimens cut in the machine direction of commercial fine paper: (⊕) single specimen, ⊠ multiple specimen.

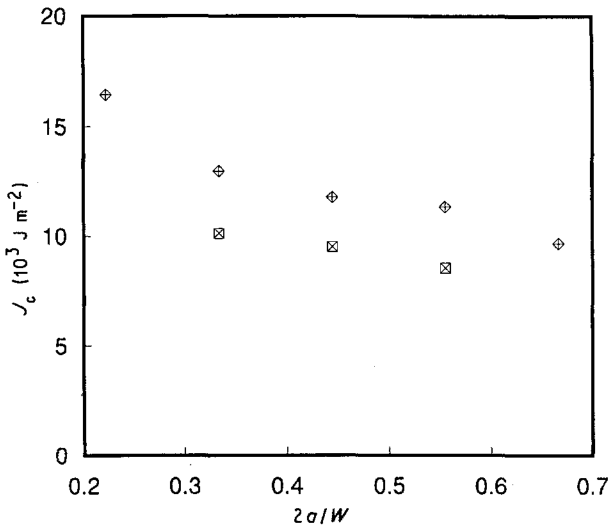

Figure 5 Critical J-value, J_c, against notch length normalized by specimen width for specimens cut in the cross-machine direction of commercial fine paper: (⊕) single specimen, (⊠) multiple specimen.

2.4. Problems with the single-specimen method

As reported by many researchers, single-specimen method estimates of the J-integral for paper do not always agree with the values obtained with the multiple-specimen method. Fig. 4 shows an example of relatively good agreement between values estimated with the two methods. These results were obtained with DEN specimens (width 90 mm, length 200 mm) cut in the machine direction (MD) of fine paper made from fully bleached kraft pulp. In Fig. 5, however, relatively poor agreement between the two methods is illustrated for the specimens cut in the cross-machine direction (CD) of the same material (note that paper is an orthotropic material and that the properties of MD and CD specimens are expected to be different). For these specimens, the single-specimen method gives results for J_c which are consistently high. Since the only difference between the data of Figs 4 and 5 is the orientation of the specimens with respect to the machine direction, the accuracy of the single-specimen method is clearly dependent on the properties of the sample sheet. The evaluation of J_c for paper using the single-specimen method is therefore questionable.

2.5. Previous approaches

There have been many articles discussing the accuracy of the single-specimen method in estimating the J-integral for materials which display both small- and large-scale yielding prior to fracture [15, 18, 22–38]. In these studies, the derivation of simplified equations for estimating the J-integral were based on modifications of the assumption about the P–δ_p relationship first proposed by Rice *et al.* [8], together with various assumptions for the constitutive equation of the material.

By analogy to the elastic-energy release rate, G, Sumpter and Turner [29] suggested that the J-integral can be expressed as follows

$$J = \frac{1}{b}(\eta_e U_e + \eta_p U_p) \qquad (10)$$

where U_e and U_p are the elastic and the plastic component of the work per unit thickness done on a specimen, respectively, which are estimated as areas under a load–displacement curve, and η_e and η_p are so called eta-factors (η-factors), which are dimensionless functions of geometrical parameters such as the crack length to specimen width ratio. Paris *et al.* [30] investigated the eta-factor for specific specimen geometries. In the ASTM Standard Test Methods for J_{Ic} (E 813) [31] and J–R curves (E 1152) [32], the following forms are used:

$$\eta_p = 2$$

for an edge-notched three-point bend specimen, and

$$\eta_p = 2 + 0.522\frac{b}{w}$$

for a compact tension specimen. Recently, Sharobeam and Landes [33] proposed a novel method for experimentally determining the η_p-factor, involving measurements of the load–displacement curves for specimens with differing crack lengths, and then

Sharobeam *et al.* suggested [34] that previous η values may not be entirely correct.

On the other hand, a two η-factor description for the plastic part of the J-integral has been discussed by several authors [18, 26, 35–38]. This description is based on an expression using the energy and the complementary energy integral estimated with the load–displacement (plastic component) curve, as follows

$$J_p = \frac{1}{b}\left[\eta_r \int_0^{\delta_p} P\, d\delta_p + \eta_c \int_0^{P} \delta_p\, dP\right] \quad (11)$$

where η_r and η_c are dimensionless factors, which are functions of geometrical parameters only. The plastic component of displacement has been expressed in the following form

$$\delta_p = \varphi(b) H[P\,\psi(b)] \quad (12)$$

where φ and ψ are normalizing functions with the same dimensions as δ_p and P, respectively. The η-factors, η_r and η_c are then given by

$$\eta_r = -\frac{b}{\psi(b)}\frac{d\psi(b)}{db} \quad (13)$$

$$\eta_c = -\frac{b}{\varphi(b)}\frac{d\varphi(b)}{db} \quad (14)$$

Recently, Smith [38] showed further solutions of η-factors for compact tension specimens meeting the ASTM Standard for J–R testing (E 1152) [32] for both small-scale yielding and large-scale plasticity. For the DEN specimen, McMeeking [26] obtained the following approximation from finite-element simulations of strain-hardening materials having n-values of 5–20 and notch-length/specimen-width ratios (a/w) of 1/4–5/8

$$\eta_r = \frac{0.31}{2.22 - 1.60a/w}$$

$$\eta_c = \frac{b}{2a}$$

Unfortunately, McMeeking's assumptions have not been investigated theoretically or experimentally for thin materials like paper.

3. Analysis

3.1. The origin of the discrepancy between the single-specimen method and the multiple-specimen method

In this study, the discussion begins with the assumptions first presented by Rice *et al.* [8] in the original paper on the single-specimen method. The discrepancy between the single- and multiple-specimen methods must result from one of the two assumptions used to derive the single-specimen J-integral. The first assumption, expressed in Equation 3, is purely phenomenological and could not give rise to the error. The second assumption, as expressed in Equation 4, does have a restriction, however: the function h must be a single function of P/b.

In order to investigate the function h, the relationship between δ_p/b and P/b were plotted in Figs 6 and 7 with the same load–displacement curves used for the J-integral evaluations in Figs 4 and 5, respectively. The values of P/b were normalized by σ_y (the yield stress of an unnotched specimen) so that the two graphs could be plotted on the same scale.

For the specimens cut parallel to the machine direction of fine paper, the relationship between δ_p/b and P/b appears to be relatively independent of crack length, indicating that the assumption that h is a single function of P/b is valid. For these specimens, the results of the single- and multiple-specimen methods were in agreement (Fig. 6).

For the specimens cut with their long axis aligned in the cross-machine direction, the relationship between δ_p/b and P/b appears to be dependent on crack length, indicating that the assumption that h is a single func-

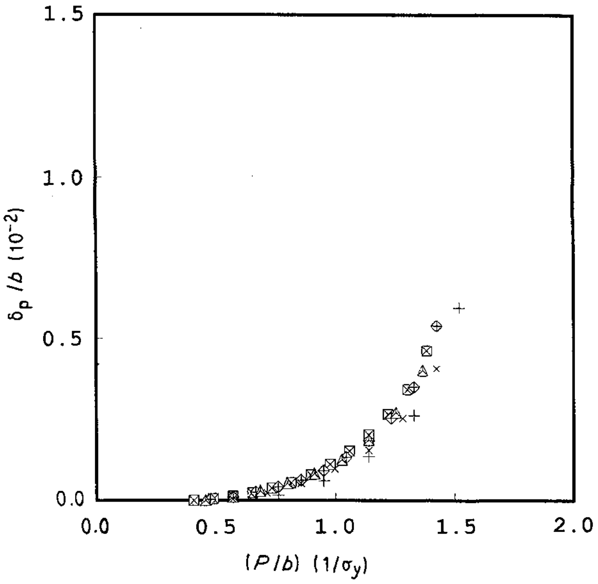

Figure 6 Relationship between the displacement due to plasticity and the normalized load for specimens cut in the machine direction of commercial fine paper. Single notch length: (⊠) 10 mm, (⊕) 15 mm, (△) 20 mm, (×) 25 mm, (+) 30 mm.

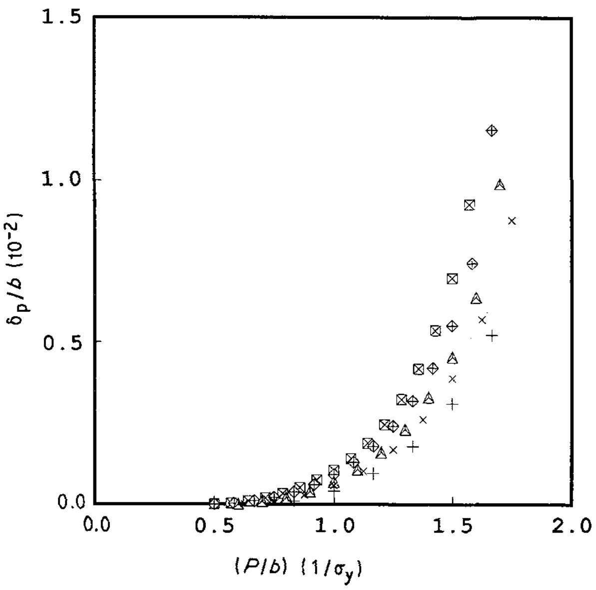

Figure 7 Relationship between the displacement due to plasticity and the normalized load for specimens cut in the cross-machine direction of commercial fine paper. Key: as for Fig. 6.

tion of P/b is not valid for these specimens. The results of corresponding J-integral tests show disagreement between the single- and multiple-specimen methods. It is clear that the discrepancy in the J-integral is a result of the fact that the function h is not unique.

Similar results showing a separation of the P–δ_p curves as a function of notch length were shown in the finite-element simulation of fully yielding elastic–plastic specimens performed by McMeeking [26], Hoshide *et al.* [25] and Hayashi *et al.* [28].

3.2. Generalized assumption for the displacement due to plasticity

Consider a slight modification of Rice's assumption regarding the relationship between the displacement due to plasticity, δ_p, and the specimen geometry. It is assumed that the relative influences of the load, P, and the ligament length, b, on the plastic displacement, δ_p, are related in some non-linear way and this may be expressed as

$$\delta_p = b\,h(P/b^m) \qquad (15)$$

where h is a non-dimensional function of w, σ_y, and (P/b^m).

Fig. 8 shows several plots of δ_p/b against P/b^m with various values of m for the specimens taken from the cross-machine direction of fine paper (the same specimens are represented in Figs 5 and 7). When m is equal to 0.8, the lines for all ligament lengths are coincident. In this case, it appears that h is a single valued function of $P/b^{0.8}$.

3.3. Extended method of J-integral estimation

Using the new assumption expressed by Equation 15, the integrand in Equation 5, for the plastic part of the J-integral, becomes

$$-\frac{\partial \delta_p}{\partial b} = m\frac{P}{b^m}h'(P/b^m) - h(P/b^m)$$
$$= \frac{1}{b}\left[mP\left(\frac{\partial \delta_p}{\partial P}\right)_b - \delta_p\right] \qquad (16)$$

Substituting Equation 16 into the second term of Equation 5 and integrating by parts gives

$$J = J_e + \frac{1}{b}\left[(1+m)\int_0^{\delta_p} P\,\mathrm{d}\delta_p - P\delta_p\right] \qquad (17)$$

The elastic part of the deformation, δ_e, can be expressed using the initial slope, C, of the load–displacement curve [19, 4], as

$$\delta_e = CP \qquad (18)$$

Therefore, from Equation 3, δ_p can be expressed as

$$\delta_p = \delta - CP \qquad (19)$$

Substituting Equation 19 into Equation 17, the final expression of the modified J-integral is obtained as

$$J = J_e + \frac{1}{b}\left[(1+m)\int_0^{\delta} P\,\mathrm{d}\delta - P\delta + \frac{(1-m)}{2}CP^2\right] \qquad (20)$$

When m has a value of one, Equation 17 is identical to the simplified equation representing the single-specimen method of Rice *et al.* i.e. Equation 6.

Using Equation 20, the J-integral can be estimated more accurately from single-specimen data than was previously possible. In order to use this equation, the parameter m and the initial slope, C, are needed for each material. The parameter m is believed to be a property of the material being tested and can be estimated with (a minimum of) two specimens with differing notch lengths by plotting δ_p/b against P/b^m and using a least-squares fit to an appropriate function representing h. The initial slope, C, is obtained from the load–displacement curve for the specimen being tested.

4. Results and discussion

Fig. 9 shows the variation of J values with respect to the displacement in both machine and cross-machine directions for commercial fine paper. The values of J were evaluated using four separate methods: the multiple-specimen method, the single-specimen method, the method based on the McMeeking approximation, and the authors' proposed method. For specimens cut in the machine direction, the differences in the results produced by the four methods were not significant. However, for specimens cut so that their long axes were aligned with the cross-machine direction, the authors' method and the multiple-specimen method (the most "correct" method) gave very similar results, while the single-specimen method gave values which were relatively high. The results based on McMeeking's approximation were very close to those of the multiple-specimen method except near the critical displacement.

Fig. 10 shows the variation of J_c determined by the four methods described above with respect to the ratio of notch length to specimen width, for specimens of commercial fine paper cut in both the machine and cross-machine directions. Because of the difficulty in defining the onset of crack growth in paper specimens, the value of the J-integral at the maximum load point was taken to be J_c in this study.

The single-specimen method yielded larger J_c values than the multiple-specimen method for every notch length. The values estimated using the method based on the McMeeking approximation and the authors' proposed method, however, matched those obtained by the multiple-specimen method very closely.

It should be noted that none of these methods give a value of J_c which is truly geometry independent. This is most likely because of the use of the maximum-load point instead of the point of the true onset of crack initiation, but at present there is no alternative to this technique. In spite of this shortcoming, a failure criterion based on a critical value of the J-integral is still believed to be the best method for predicting the onset of fast fracture in paper. Although there have been many measurements of the work of fracture during slow crack growth, the potential application of these results to the prediction of unstable fracture in

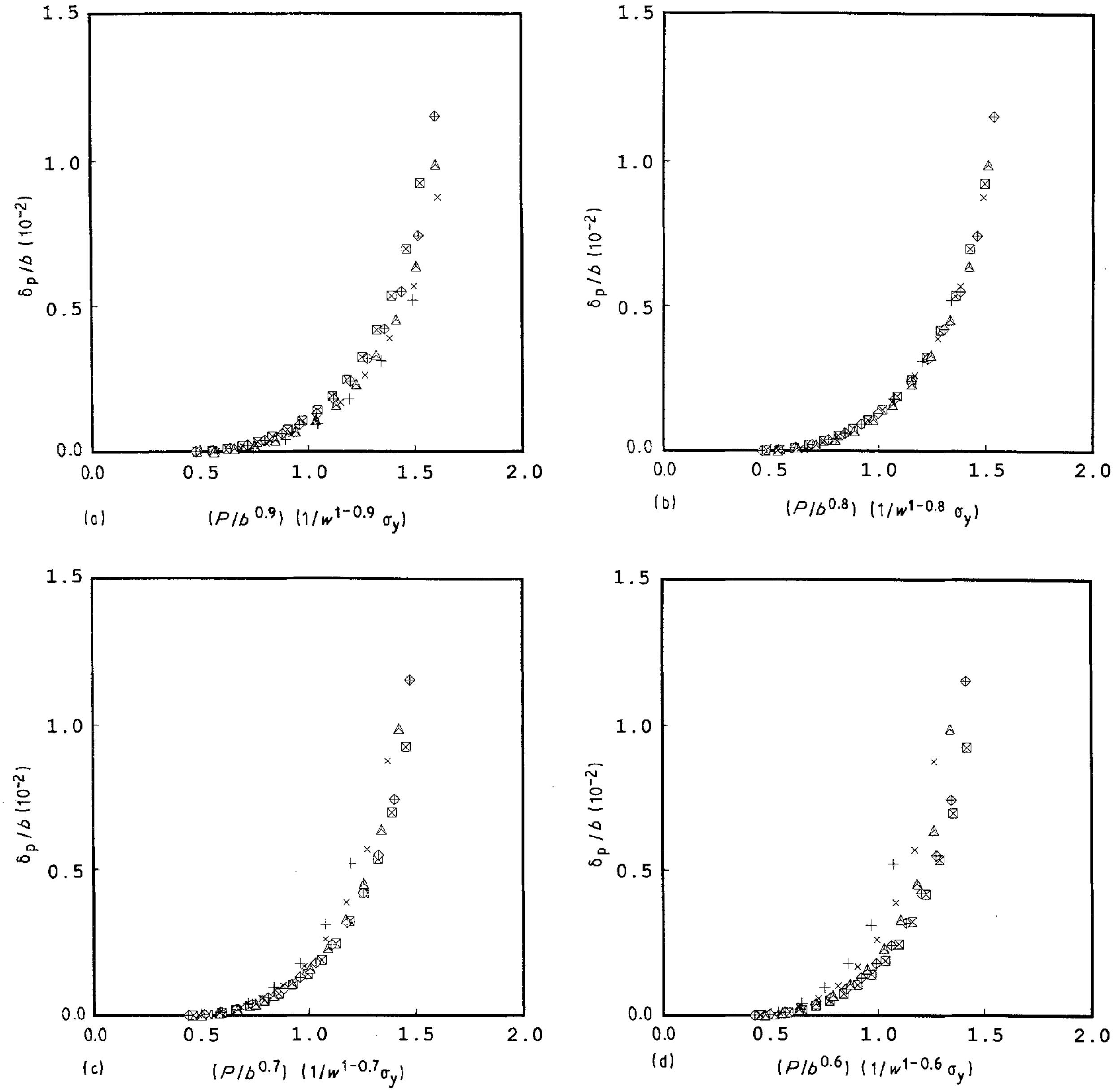

Figure 8 Relationship between the displacement due to plasticity and the normalized load, $(P/b^m)(1/w^{1-m}\sigma_y)$ for specimens cut in the cross-machine direction of commercial fine paper: (a) $m = 0.9$, (b) $m = 0.8$, (c) $m = 0.7$, and (d) $m = 0.6$. Key: as for Fig 6.

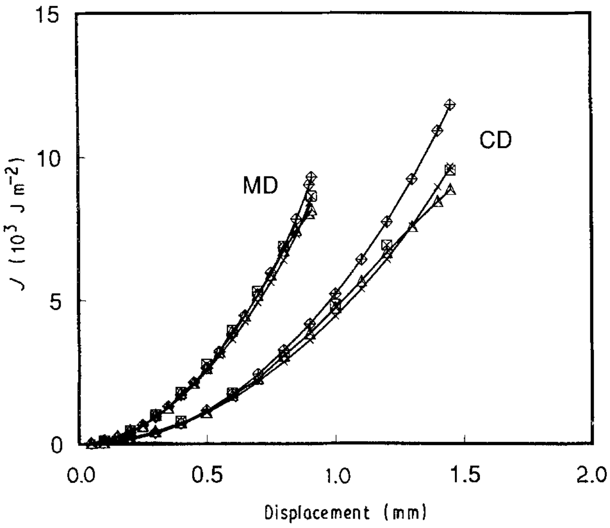

Figure 9 J-integral versus crosshead displacement for commercial fine paper. (×) Authors' method (machine direction (MD), $m = 0.9$. Cross-machine direction (CD), m = 0.8), (⊕) single specimen, (⊠) multiple specimen, and (△) McMeeking's approximation.

practical applications such as high-speed printing presses is not well understood.

Fig. 11 shows another example of the application of the authors' proposed method, obtained using commercial bond paper. J_c values estimated with the method based on the McMeeking approximation and the authors' method are in agreement with the results from the multiple-specimen method here as well. However, the method based on the McMeeking approximation gives erroneously high values of J_c for short crack lengths ($2a/w = 0.33$). The accuracy of the method based on the McMeeking approximation for estimating the J-integral of paper DEN specimens must depend on specimen geometry and material properties. In contrast to this, the authors' proposed method provides good agreement with the multiple-specimen method for all materials and geometries tested to date. In spite of the fact that at least two specimens are required to evaluate the parameter m for a new material, the new method is much simpler

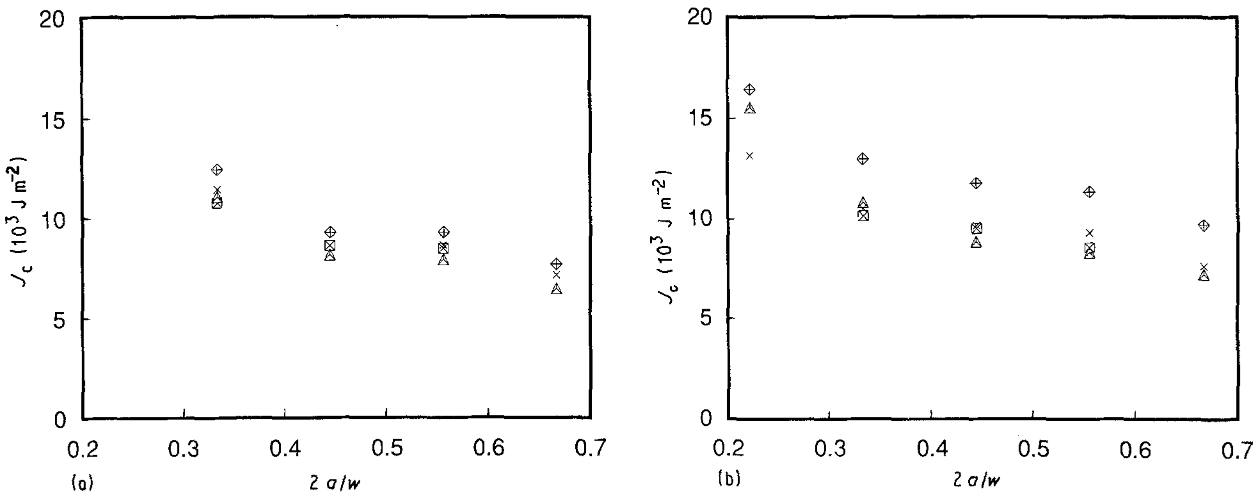

Figure 10 Critical J value, J_c, versus notch length normalized by specimen width of commercial fine paper: (a) machine direction, and (b) cross-machine direction. (×) Authors' method ($m = 0.9$), (⊕) single-specimen method, (⊠) multiple-specimen method, and (⚠) McMeeking's approximation.

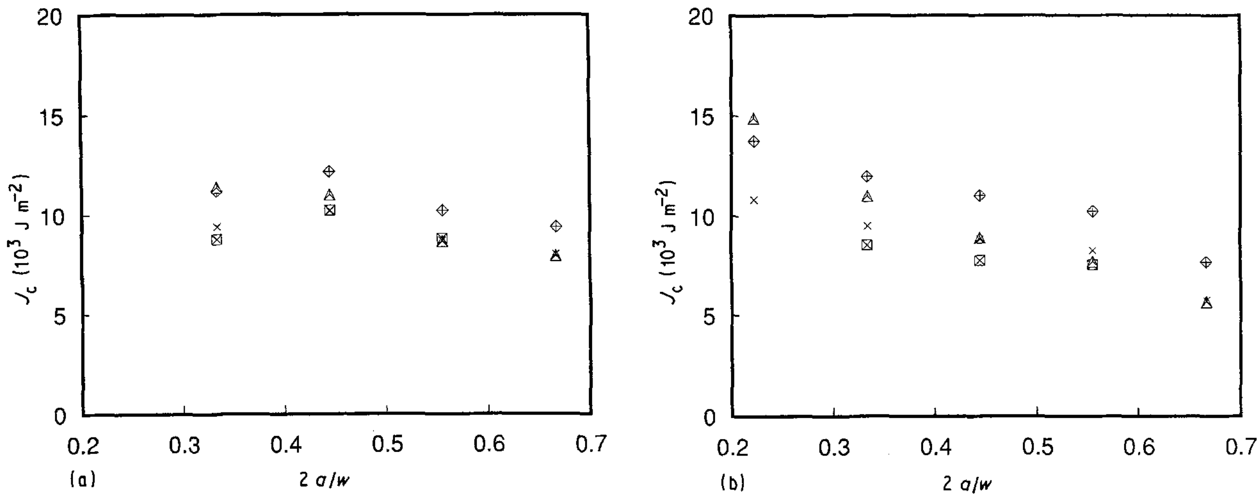

Figure 11 Critical J value, J_c, versus notch length normalized by specimen width of commercial bond paper. (a) Machine direction, (b) cross-machine direction: (×) Authors' method ($m = 0.8$), (⊕) single-specimen method, (⊠) multiple-specimen method, (⚠) McMeeking's approximation.

and requires far fewer specimens than the multiple-specimen method.

Recently, Westerlind *et al.* [21] showed that the critical values from the Liebowitz non-linear technique, which is similar in some respects to the J-integral approach, gave results very close to those obtained with the multiple-specimen method and they were also notch-length independent. However, the results presented here demonstrate that an evaluation of the J-integral based on the original definition and a simple assumption can also provide very accurate results.

Comparing the authors' simplified equation (Equation 17) with the two η-factor description, the following interpretations for the functions φ and ψ in Equation 12 are possible:

$$\varphi(b) = b$$
$$\psi(b) = b^{-m}$$

Therefore, η_r and η_c in Equation 11 can be expressed using Equations 13 and 14 as

$$\eta_r = m$$
$$\eta_c = -1$$

Equation 17 can be derived easily from Equation 11 using the above values for η_r and η_c.

In this study, it was assumed that no latent stable crack growth occurred prior to the critical displacement. In practice, however, the fracture of notched paper specimen is typically preceded by some stable crack growth, particularly for specimens cut in the cross machine direction. It is possible that this crack growth, which is certainly a function of the material properties, leads to the relationship between the parameter m and the material properties. This relationship could be further explored with finite-element simulations.

TABLE I Mechanical properties of sample materials. Mean values and 95% confidence limits obtained from 10 tests are listed: Yield stress is a 0.2% offset yield stress, MD is the machine direction, and CD is the cross-machine direction

Specimen	Grammage ($g\,m^{-2}$)	Thickness (mm)	Elastic modulus (GPa)		Tensile strength ($kN\,m^{-1}$)		Ultimate strain (%)		Yield stress ($MN\,m^{-2}$)	
			MD	CD	MD	CD	MD	CD	MD	CD
Fine paper	72.0	0.090	6.66 ± 0.11	3.05 ± 0.03	3.40 ± 0.11	2.12 ± 0.08	1.67 ± 0.12	4.61 ± 0.37	23.5	13.4
Bonds paper	88.7	0.111	7.82 ± 0.10	2.56 ± 0.03	4.71 ± 0.13	2.14 ± 0.05	1.50 ± 0.08	4.09 ± 0.22	24.5	9.89

5. Conclusion

It has been demonstrated that the inaccuracy of the previously defined single-specimen method when applied to the fracture-toughness testing of paper results from the erroneous assumption that δ_p/b may be expressed as a single function of P/b. This assumption was shown to be false for paper specimens, but a simple extension of the assumption was found to be adequate to model the behaviour of paper. A novel method for estimating the J-integral, based on the extended assumption, was developed analytically. Although the new method requires at least two specimens in order to evaluate a new material parameter, it then provides more flexibility than previous modified simplified equations which had been developed primarily for metallic materials. The experimental study indicated that the present method produces values of J_c which agree very closely with those obtained using the cumbersome multiple-specimen method, and is therefore more suitable for paper testing than other methods presently available.

Appendix: Experimental procedure

Specimens were cut from the machine and cross-machine directions of fine paper made from fully bleached hardwood kraft pulp and bond paper made from fully bleached softwood kraft pulp. The mechanical properties of the test samples are summarized in Table I. All tests were performed at 23 °C and 50% relative humidity. Notched (DEN) specimens (90 mm wide) were used and the specimens contained edge notches of length 10, 15, 20, 25 and 30 mm.

A Sintech 1 tensile tester was equipped with 100 mm wide grips rigidly attached to the frame. The initial distance between the grips was 200 mm. The crosshead speed was $2\,mm\,min^{-1}$ for all measurements. The load–displacement curves were recorded and analysed on a personal computer. A typical load–displacement curve for a notched commercial fine-paper specimen is presented in Fig. 1. Machine compliance was subtracted from all load–displacement data.

For each notch length, the load–displacement curves for six specimens were averaged together, and the averaged curves were used for the J-integral computation.

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